Details, Discoveries, and Anomalies of 3-Space Positive 4-Axes Coordinate Systems

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Early in the exploration of iterative equation complex structures in 3-space it became apparent that there were massive amounts of new data being generated. The programs were designed to be flexible and as general as possible - the result was too much data all at once to process. That is the reason these draft quality papers are being presented on the web: I don't have the expertise in the many areas of mathematics, let alone physics, chemistry, etc., to properly explain the findings of the programs I've written. Sprinkled liberally throughout this paper you'll find the parenthetical, '(Why?)', following empirical statements, indicating yet unresolved questions. If you can answer any of these queries I'd be delighted to hear from you. Actually, any additions, corrections, or suggestions are encouraged.

This paper deals exclusively with the complex 3-space positive 4-axes coordinate system. The four axes are arranged in a tetrahedral fashion which may have some effect on the structures formed by the iteration equations. Almost certainly it will help to explain the occurrence of the hexagonal close-packing number (HCP) in many of the structures. This may present a new and useful tool for crystallography.

While designing and coding the programs I used one structure repeatedly to check for correct functioning after modifying the code, referred to as the 'flagship', as it displays many of the characteristics discussed in this paper. (See image below.)

The image, as explained in the previous paper on this web site, is a slice through the 3D structure, in this case the slice is the XZ plane. The black color indicates the set of those points whose positive axes addresses, each taken as a complex-like number, stayed finite when iterated. The iteration equation (sometimes referred to as the formula) used was a cubic:

$$z_{(n+1)} = z_n^3 + z_n^2 - z_n \ , \quad z = r + y + g + b \ ,$$

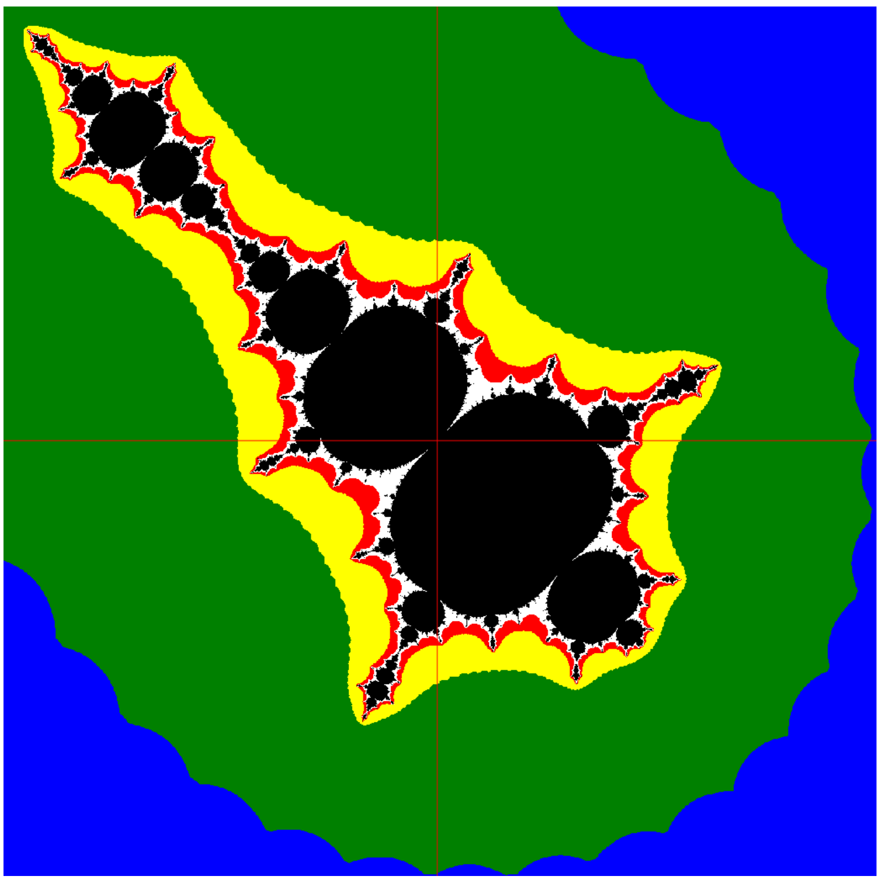
where r, y, g, and b are the 4 coordinates of the address. The algebra used is 'algebra 2' defined in a previous paper (Complex Positive Axes Systems) and mimics the standard complex plane algebra.

The colored areas indicate 'how fast' the points in those areas 'go to infinity'. Actual practice makes use of several factors indicating the iteration will not stay finite. In this case a point in the white area required at least nine iterations to be flagged as not in the set; red needed only 7 or 8 iterations to find it would be infinite, yellow 5 or 6, green 3 or 4, and blue only 1 or 2. There are scallops at the interface between colors. (Why?)

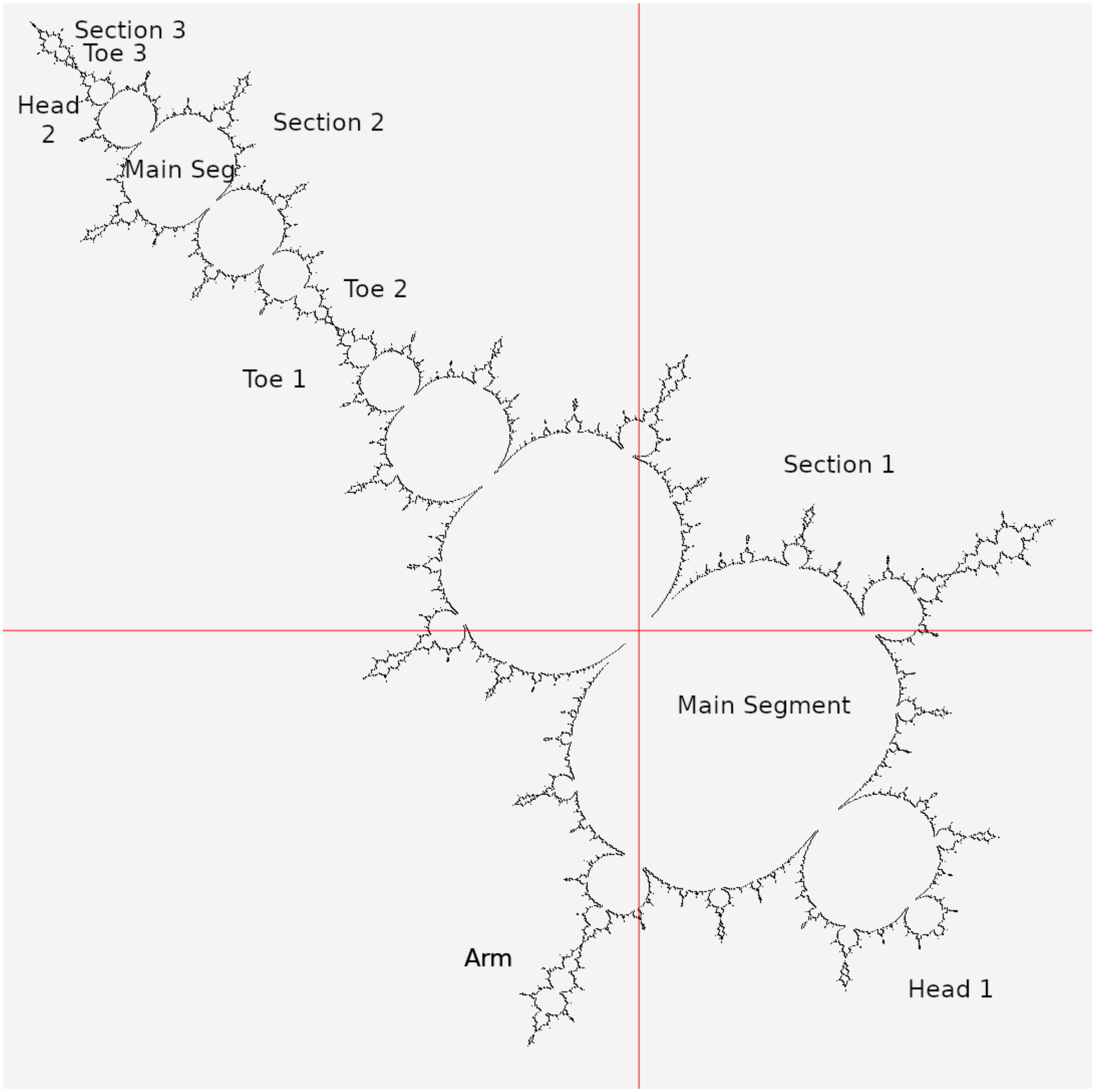
The red cross hair lines in this image are the X axis (horizontal) and Z axis (vertical), indicating the origin at their intersection. These cross hairs actually indicate a programmatic start point for calculations and are often not on the origin. The distance the XZ plane is moved along the Y axis in this image is zero, so, in this image the true XYZ origin is indicated by the cross hairs.

The length of the entire set (black figure) is 2.449... , and the length of just the largest section is 2/3 of the entire length making its length 1.632... , which is the hexagonal close-packing number, the HCP, of √(8/3). (Why?) (The Mandelbrot set also displays the HCP, but only when displayed in this space.) The length of the flagship is fixed, but both ends of the structure are infinite regressions. On the upper end of the image the sections endlessly repeat getting smaller. The other end endlessly repeats smaller round balls.

In order to make explanations easier an image with labels follows the flagship.



The flagship



The flagship with various parts labeled

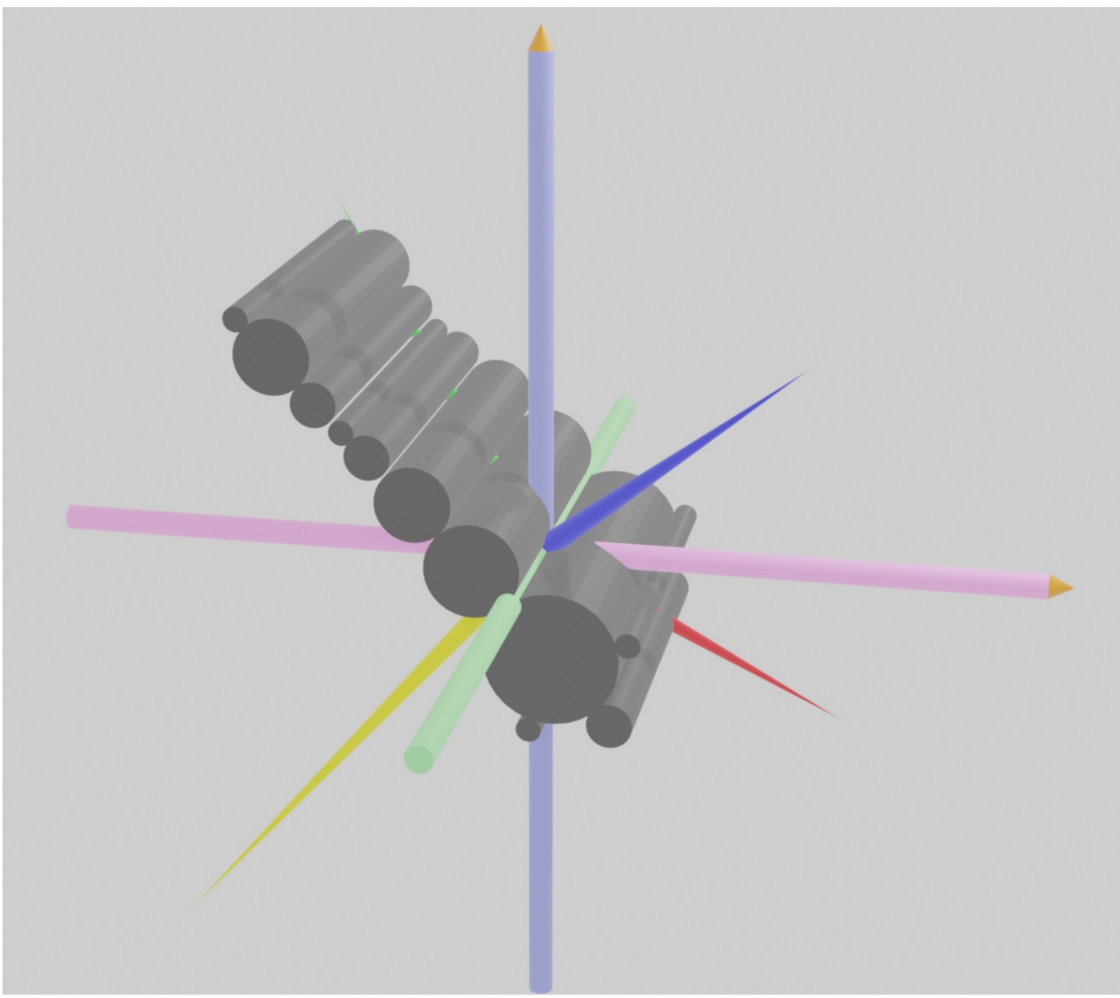
The structure has an infinite number of sections, and each section has an infinite number of segments. The first section is quite different from all the rest. Section 1 straddles the origin. It is ‘head down’ while all the others are ‘heads up’. (Why?)

The ratios of adjacent section lengths form a convergent sequence that converges to 1/7. (Why?) Each section after the first few is 7 times larger than the following section. This rapid convergence is much like the Feigenbaum convergence constants for periodic doubling. The sections could reflect some type of bifurcation with a constant of 7. But there are other convergent sequences as well in the structure of the flagship.

The largest segment in each section is labeled the main segment. Looking at section 1, the ratio of the size measured longitudinally of the main segment to the adjacent segment nearer the head is 2.277... and the ratio of sizes of the segment 1 from the main segment and segment 2 from the main segment towards the head is 3.458... The ratios converge to 4 rather rapidly. (Why?) Going the other way, towards the toe of the first section and again starting with the main segment, the ratios converge to 2. (Why?)

Each of the other sections differ from the first section in that their ratios, from each of their main segments, converge on a value of 2 in both directions, towards the head and towards the toe. (Why?) Again, these convergences seem to mirror what Feigenbaum discovered. Each section may represent a periodic doubling of some sort.

So far, we have only looked at a single slice through the flagship. Before showing the images of other slices it is far easier to first display a very rough image of what the entire structure looks like; a mock up done in Blender.



The flagship in 3D (Blender)

The Cartesian axes are, as always, X pink, Y green, and Z lavender. The Red, Yellow, Green, and Blue positive 3-space axes are shown in the proper orientation. This shows that the 3D structure is not a rotation of the 2D, but an extrusion. (Why?) It also shows why we get a ‘flat’ cross section when we use the XZ plane to slice the structure. The flagship angles upwards at a 45 degree angle as it stretches into the negative X space. (Why?) The lengths of the columns that are the extrusions of the segments are all equal and measure 1.732... , which is the square root of 3. (Why?)

The 3D image extends to both sides of the XZ plane and it turns out 1/3 of the structure is on the positive Y side of the XZ plane and 2/3 is on the negative side. (Why?)This can be seen in the image of the flagship sliced by the YZ plane:

The flagship sliced by the YZ plane

The Y axis is the horizontal red line, and the Z axis is the red vertical line.

The program producing these images allows for many variations: the factors of each term in the iteration formula can be changed, a ‘constant’ term (the original point address, as in the Mandelbrot iteration formula) can be added, a different algebra can be used, etc. These structures have been briefly examined and seem to have much more to inform us.

Next, we will examine the actual logistic map iteration formula in the 3-space positive 4-axes system.