Positive Axes Systems

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There are several alternative axes systems that can be used to examine data sets generated by various functions, including iteration formulas such as the one used to create the Mandelbrot set.

In 2-space and 3-space we generally use a Cartesian coordinate system. The axes form right angles with each other, and each axis has positive numbers in one direction from the origin and negative numbers in the other direction. It is necessary for the axes taken together to both span the space and be independent of each other, that is, every point in the space must be uniquely addressable (spans) by a summation of distances from the origin along some or all of the axes, and no point on a single axis can be addressed by a combination of the other axes (independence). The Cartesian system certainly satisfies these constraints, but there are other systems that do, as well.

In all of the alternative systems examined here, the axes are rays, not lines, starting at the origin and continuing ‘without limit’. The units along the axes are always positive. Additionally, the axes are always arranged symmetrically about the origin, although, of course, the angles between the axes are no longer 90 degrees. The number of positive axes that can both span the space and be independent range from n+1 to 2*n, where n is the spacial dimension. Interestingly, in 3-space the least number of positive axes is 4, while in 2-space the largest number of positive axes is 4. This is what allows a complex coordinate system to be expressed in 3-space. (See paper ‘Complex Positive Axes Systems’.) The angles between adjacent axes in positive systems varies between 90 degrees for the largest number of positive axes, and the inverse cosine of negative 1/n degrees for the smallest number of positive axes, where n is the spacial dimension. Thus the positive 3-axes in 2-space are 120 degrees apart, and the positive 4-axes in 3-space are about 109.47 degrees apart. In 1-space (a line) there can be 2 positive axes 180 degrees apart.

(A note on nomenclature: in this paper an axes system that has positive and negative values along perpendicular axes is labeled ‘Cartesian’. An axes system that uses positive only values along any number of rays from the origin is labeled ‘positive’. This paper only deals with the minimum number of positive axes needed to independently span the space being studied, that is, n+1.)

An example of one of these positive systems in 2-space reuses the Cartesian system with the stipulation that there are four axes instead of two, and that what were the negative halves of Cartesian axes become positive axes in their own right. This system of axes certainly spans the plane and the axes are independent – as long as no negative address coordinates are allowed. In order to resolve negative results from mathematical calculations and to retain unique addresses, an altered algebra for addresses must be in place. The address of a point in this 2-space must have at least two zeros as part of the four coordinates. Labeling the four axes X, Y, S, andT, with S in the place that was occupied by negative X and t in place of negative Y, we see that if a point with an address such as (1,2,3,4) were to be placed on the grid it would be at (0, 0, 2, 2). Computationally, if two ‘opposed’ axes both have non-zero address values the smaller can be subtracted from itself and from the larger of the two values. This process is called normalization, and as we’ll see, various alternative coordinate systems all require some type of algebraic normalization. After normalization all the herein described coordinate systems meet both tests for functional address systems: they span the space and are independent.

But four is not the only number of positive axes that independently span the plane; three such axes do so quite nicely. Consider three axes identified by color, R (red) aligned with the Cartesian positive X axis, G (green) 120 degrees counter clockwise from R, and B (blue) counter clockwise 120 degrees from G. For this system of axes to work we need an algebra that normalizes addresses to at least one zero coordinate. Given three non-zero coordinates in an address such as (1, 2, 3), we subtract the smallest from all three coordinates yielding (0, 1, 2) as the single unique normalized address. Subtracting the smallest from all coordinates isn't an arbitrary decision, it is driven by the law of cosines (which for the Cartesian systems is just the Pythagorean theorem). We’ll see a similar ‘natural’ algebra in the 3-space realm.

In this system of 3-axes in 2-space there is a direct mapping from the Cartesian plane. Any function that can be graphed in a Cartesian system can be graphed in a positive axes system.

Moving on to 3-space we have standard three axes in the Cartesian system, but we can transform that system into positive 6-axes in a manner similar to what we did in 2-space. It turns out that 4-axes and 5-axes systems can also be formed that span the 3-space and are independent. The 4-axes system is the one we examine here. The axes are symmetrically placed and form the spokes (rays) from the origin to the vertices of a tetrahedron. This means the axes are about 109.5 degrees apart. We’ll label them red, yellow, green, and blue (R, Y, G, B). Among the unlimited number of possible alignments relating the Cartesian system with the positive axes system, the only difference among them is the programmatic ease of translating coordinates between the two. The orientation shown below is used throughout this series of papers. The Cartesian system axes are: X, pink; Y, light green, and Z, lavender. The positive direction of each is indicated by a small cone on the end of the axis. The positive axes are: R, red; Y, yellow; G, green; and B, blue.

The algebras for these 3-space 4-axes positive systems is similar to that of the 2-space 3-axes positive axes system. At least one coordinate must be zero and (surprising in its simplicity) subtracting the smallest from all four coordinates normalizes the address. Again there are straight forward mappings from Cartesian address in 3-space to the positive 4-axes address in 3-space. This 3-space 4-axes positive system is tetrahedral in shape and may have applications in chemistry, crystallography, and particle physics, but those applications are not dealt with here.

Positive axes systems become much more interesting when a complex algebra is used with these positive axes systems, as described in the next part of this paper.